

Effective nucleon-nucleon interactions and nuclear matter equation of state

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Received: 5 May 2000

Communicated by A. Molinari

Abstract. Nuclear matter equations of state based on Skyrme, Myers-Swiatecki and Tondeur interactions are written as polynomials of the cubic root of density, with coefficients that are functions of the relative neutron excess δ . In the extrapolation toward states far away from the standard one, it is shown that the asymmetry dependence of the critical point (ρ_c, δ_c) depends on the model used. However, when the equations of state are fitted to the same standard state, the value of δ_c is almost the same in Skyrme and in Myers-Swiatecki interactions, while is much lower in Tondeur interaction. Furthermore, δ_c does not depend sensitively on the choice of the parameter γ in Skyrme interaction.

PACS. 21.65.+f Nuclear matter – 21.30.Fe Forces in hadronic systems and effective interactions

1 Introduction

Nuclear matter is considered as an uncharged nucleon system distributed uniformly in the space, and nuclear matter equation of state is the energy per nucleon $e(\rho, \delta)$ of nuclear matter given as function of nucleon density ρ and relative neutron excess δ . The equation of state $e(\rho, \delta)$ is a fundamental quantity in theories of neutron stars and supernova explosions, as well as in studies of nucleus-nucleus collisions at energies where nuclear compressibility comes into play [1].

The main measured quantities which can provide information about equation of state (EOS) are the binding energies and other data of finite nuclei. As the finite nuclei are in states near the standard nuclear matter state, which is the ground state of nuclear matter with normal nucleon density ρ_0 and zero neutron excess, $\delta = 0$, therefore, our knowledge about EOS can be confirmed experimentally only in a small region around $\rho \sim \rho_0$ and $\delta \sim 0$. In this region, the main quantities which specify the EOS are the coefficients a_1 (volume energy), J (symmetry energy), K_0 (incompressibility), L (density symmetry), and K_s (symmetry incompressibility). Nowadays the quantities which are known with enough precision are a_1 , J and K_0 , while the last two are still under investigation.

However, there is currently considerable interest in the very neutron-rich nuclei and the energetic heavy-ion collisions where the nuclear matter state is beyond this region. As any direct information beyond this region is difficult

to come by, extrapolation is inescapable and, in this case, a nuclear model is required. This model is fitted to binding energies and other data of finite nuclei at first, then applied to nuclear matter to derive the EOS. In this way, the obtained EOS can be considered as being fitted indirectly to a region around the standard state, but its prediction on states beyond this region should be regarded as an extrapolation. Obviously, the reliability of this extrapolation depends on the foundation of the model. In order to be reliable, the model should be based on a well-founded theory with as few adjustable parameters as possible, which are fitted to as many high-accuracy measured data as possible. Basically, this is what we understand by model of effective nucleon-nucleon interaction, for example the Skyrme [2], the Myers-Swiatecki [3] and the Tondeur interactions [4]. The EOSs derived from these effective interactions have analytical expressions which can be pictured and calculated easily, so they are used widely in the literature, even they are based on energy functional theories which are in the macroscopic level. Instead of effective nucleon-nucleon interaction, nuclear matter is studied also in more basic level by “microscopic” potentials available in the market and by sophisticated many-body theories for many years [5–9].

The purpose of the present paper is to discuss the EOSs given by Skyrme [2], Myers-Swiatecki [3] and Tondeur interactions [4], in comparison with the microscopic calculations. In section 2, the nuclear EOS given by these interactions is presented. The equilibrium condition and the properties of standard nuclear matter are discussed

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in section 3, while the predictions for nuclear matter away from the standard state are given in section 4. Section 5 makes comparison with some microscopic calculations. In section 6 a short discussion and summary are addressed. Appendix A presents a specific discussion on Myers-Swiatecki interaction and Appendix B gives some formulas used in section 4 to calculate the interaction parameters from standard nuclear matter quantities.

2 Nuclear equation of state

The nuclear energy E_N of a nucleus can be written as

$$E_N = \int d^3r \mathcal{E}_N, \quad (1)$$

where the nuclear energy density functional \mathcal{E}_N can be written with enough generality as

$$\mathcal{E}_N = \rho(\mathbf{r})e(\rho, \delta) + \mathcal{E}_{\text{GD}}, \quad (2)$$

where

$$\mathcal{E}_{\text{GD}} = \frac{1}{2}Q_1(\nabla\rho)^2 + Q_2\left[(\nabla\rho_n)^2 + (\nabla\rho_p)^2\right] \quad (3)$$

is the gradient-density-dependent term. In the above equation, $\rho = \rho_n + \rho_p$, ρ_n and ρ_p are the neutron and proton densities, respectively,

$$\delta = \frac{\rho_n - \rho_p}{\rho} \quad (4)$$

the relative neutron excess, Q_1 and Q_2 the model parameters related to the finite size and the surface effects of nuclei.

The EOS $e(\rho, \delta)$ depends on the model of interaction, while the functional \mathcal{E}_{GD} depends also on the model of nuclei. Equation (3) is exact for Skyrme and Tondeur interactions, and is approximate for Myers-Swiatecki interaction (see Appendix A). However, the functional \mathcal{E}_{GD} is irrelevant to the present discussion, since it is irrelevant to the nuclear matter property. In the following the EOS's based on Skyrme, Myers-Swiatecki and Tondeur interactions will be given.

2.1 Skyrme interaction

The EOS based on Skyrme interaction can be written as

$$e^{\text{Sk}}(\rho, \delta) = T \left[D_2^{\text{Sk}}(\delta) \left(\frac{\rho}{\rho_0} \right)^{2/3} - D_3^{\text{Sk}}(\delta) \left(\frac{\rho}{\rho_0} \right)^{3/3} + D_5^{\text{Sk}}(\delta) \left(\frac{\rho}{\rho_0} \right)^{5/3} + D_\gamma^{\text{Sk}}(\delta) \left(\frac{\rho}{\rho_0} \right)^{\gamma/3} \right], \quad (5)$$

where $\rho_0 = 3/4\pi r_0^3$, r_0 the nuclear radius constant, T an appropriate constant with dimension of energy such that

the D coefficients are dimensionless, and γ a model parameter. It is convenient to choose T as the Fermi energy of standard nuclear matter,

$$T = \frac{\hbar^2}{2m} \left(\frac{3\pi^2}{2} \rho_0 \right)^{2/3}, \quad (6)$$

where m is the nucleon mass. The D coefficients are

$$D_2^{\text{Sk}}(\delta) = \frac{3}{10} [(1 + \delta)^{5/3} + (1 - \delta)^{5/3}], \quad (7)$$

$$D_3^{\text{Sk}}(\delta) = -\frac{3}{8} \frac{\rho_0}{T} t_0 \left[1 - \frac{2}{3} \left(x_0 + \frac{1}{2} \right) \delta^2 \right], \quad (8)$$

$$D_5^{\text{Sk}}(\delta) = \frac{3}{10} \left(\frac{3\pi^2}{2} \right)^{2/3} \frac{\rho_0^{5/3}}{T} \left\{ s_1 \left[(1 + \delta)^{5/3} + (1 - \delta)^{5/3} \right] + \frac{1}{2} s_2 \left[(1 + \delta)^{8/3} + (1 - \delta)^{8/3} \right] \right\}, \quad (9)$$

$$D_\gamma^{\text{Sk}}(\delta) = \frac{1}{16} \frac{\rho_0^{\gamma/3}}{T} t_3 \left[1 - \frac{2}{3} \left(x_3 + \frac{1}{2} \right) \delta^2 \right], \quad (10)$$

where

$$s_1 = \frac{1}{4} \left[t_1 \left(1 + \frac{x_1}{2} \right) + t_2 \left(1 + \frac{x_2}{2} \right) \right],$$

$$s_2 = \frac{1}{4} \left[t_2 \left(x_2 + \frac{1}{2} \right) - t_1 \left(x_1 + \frac{1}{2} \right) \right], \quad (11)$$

and $t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, \gamma$ are the interaction parameters. It is worthwhile to note that, among these interaction parameters, only $t_0, t_3, x_0, x_3, s_1, s_2$, and γ appear in the EOS and thus are relevant to the nuclear matter properties. Beside these interaction parameters, there is another interaction parameter W_0 [2] that appears only in the coefficients Q_1 and Q_2 and thus is irrelevant to the EOS.

2.2 Myers-Swiatecki interaction

The EOS based on Myers-Swiatecki interaction can be written as [10, 11]

$$e^{\text{MS}}(\rho, \delta) = T \left[D_2^{\text{MS}}(\delta) \left(\frac{\rho}{\rho_0} \right)^{2/3} - D_3^{\text{MS}}(\delta) \left(\frac{\rho}{\rho_0} \right)^{3/3} + D_5^{\text{MS}}(\delta) \left(\frac{\rho}{\rho_0} \right)^{5/3} \right], \quad (12)$$

where

$$D_2^{\text{MS}}(\delta) = \frac{3}{10} (1 - \gamma_u) [(1 + \delta)^{5/3} + (1 - \delta)^{5/3}]$$

$$- \frac{3}{20} \gamma_u \times \begin{cases} 5(1 + \delta)^{2/3}(1 - \delta) - (1 - \delta)^{5/3}, & \text{for } \delta \geq 0, \\ 5(1 + \delta)(1 - \delta)^{2/3} - (1 + \delta)^{5/3}, & \text{for } \delta \leq 0, \end{cases} \quad (13)$$

$$D_3^{\text{MS}}(\delta) = \frac{1}{2} \alpha (1 - \xi \delta^2), \quad (14)$$

$$D_5^{\text{MS}}(\delta) = \frac{3}{10} \left\{ B_l \left[(1+\delta)^{8/3} + (1-\delta)^{8/3} \right] + B_u (1-\delta)^2 \left[(1+\delta)^{2/3} + (1-\delta)^{2/3} \right] \right\}. \quad (15)$$

In the above equations, α , B_l , B_u , γ_l , γ_u , and ξ are the interaction parameters. In addition to these parameters, there is another one a [3], the Yukawa range of force, that is irrelevant to the EOS, as it appears only in the coefficients Q_1 and Q_2 .

2.3 Tondeur interaction

The EOS based on Tondeur interaction can be written as

$$e^{\text{To}}(\rho, \delta) = T \left[D_2^{\text{To}}(\delta) \left(\frac{\rho}{\rho_0} \right)^{2/3} - D_3^{\text{To}}(\delta) \left(\frac{\rho}{\rho_0} \right)^{3/3} + D_\gamma^{\text{To}}(\delta) \left(\frac{\rho}{\rho_0} \right)^{\gamma/3} \right], \quad (16)$$

where

$$D_2^{\text{To}}(\delta) = \frac{3}{10} \left[(1+\delta)^{5/3} + (1-\delta)^{5/3} \right] + \frac{\rho_0^{2/3} c}{T} \delta^2, \quad (17)$$

$$D_3^{\text{To}}(\delta) = -\frac{\rho_0 a}{T}, \quad (18)$$

$$D_\gamma^{\text{To}}(\delta) = \frac{\rho_0^{\gamma/3} b}{T}. \quad (19)$$

In the above equations, a , b , c , and γ are the interaction parameters. In addition, there are another two interaction parameters d and η [4], that are irrelevant to the present discussion as they appear only in Q_1 and Q_2 .

3 Standard nuclear matter

The EOS given in the last section can be written generally as

$$e(\rho, \delta) = T \left[D_2(\delta) \left(\frac{\rho}{\rho_0} \right)^{2/3} - D_3(\delta) \left(\frac{\rho}{\rho_0} \right)^{3/3} + D_5(\delta) \left(\frac{\rho}{\rho_0} \right)^{5/3} + D_\gamma(\delta) \left(\frac{\rho}{\rho_0} \right)^{\gamma/3} \right]. \quad (20)$$

The equilibrium condition $\partial e / \partial \rho|_0 = 0$, by which the standard state $\rho = \rho_0$ at $\delta = 0$ is defined, gives the following relationship among $D_2(0)$, $D_3(0)$, $D_5(0)$, $D_\gamma(0)$ and γ :

$$2D_2(0) - 3D_3(0) + 5D_5(0) + \gamma D_\gamma(0) = 0. \quad (21)$$

The 5 quantities of nuclear matter a_1 , K_0 , J , L , and K_s can be expressed as

$$a_1 = -e(\rho_0, 0) = -\frac{T}{3} [D_{20} - 2D_{50} - (\gamma - 3)D_{\gamma 0}], \quad (22)$$

$$K_0 = 9\rho_0^2 \frac{\partial^2 e}{\partial \rho^2} \Big|_0 = T [-2D_{20} + 10D_{50} + \gamma(\gamma - 3)D_{\gamma 0}], \quad (23)$$

$$J = \frac{1}{2} \frac{\partial^2 e}{\partial \delta^2} \Big|_0 = T [D_{22} - D_{32} + D_{52} + D_{\gamma 2}], \quad (24)$$

$$L = \frac{3}{2} \rho_0 \frac{\partial^3 e}{\partial \rho \partial \delta^2} \Big|_0 = T [2D_{22} - 3D_{32} + 5D_{52} + \gamma D_{\gamma 2}], \quad (25)$$

$$K_s = \frac{9}{2} \rho_0^2 \frac{\partial^4 e}{\partial \rho^2 \partial \delta^2} \Big|_0 = T [-2D_{22} + 10D_{52} + \gamma(\gamma - 3)D_{\gamma 2}], \quad (26)$$

where

$$D_{i0} = D_i(0), \quad D_{i2} = \frac{1}{2} \frac{\partial^2 D_i}{\partial \delta^2} \Big|_0, \quad i = 2, 3, 5, \gamma. \quad (27)$$

Relation (21) is used in obtaining eqs. (22) and (23), from which the following formulas can be derived:

$$K_0 = 15a_1 + [3D_{20} + (\gamma - 5)(\gamma - 3)D_{\gamma 0}]T, \quad (28)$$

$$K_0 = 3\gamma a_1 + [(\gamma - 2)D_{20} - 2(\gamma - 5)D_{50}]T. \quad (29)$$

The specific discussion for Skyrme, Myers-Swiatecki and Tondeur interactions will be given in the following.

3.1 Skyrme interaction

For Skyrme interaction, the following relationship can be obtained from eq. (21):

$$\frac{9}{8} \frac{\rho_0 t_0}{T} + \frac{\gamma}{16} \frac{\rho_0^{\gamma/3} t_3}{T} + 3 \left(\frac{3\pi^2}{2} \right)^{2/3} \frac{\rho_0^{5/3}}{T} (s_1 + \frac{1}{2}s_2) + \frac{6}{5} = 0. \quad (30)$$

Therefore, among 7 parameters t_0 , t_3 , x_0 , x_3 , s_1 , s_2 , and γ , only 6 of them are free. Considering this relation, it can be shown that a_1 , K_0 , J , L and K_s are independent of each other, in the Skyrme EOS.

A relation connecting t_3 to a_1 and K_0 can be obtained from eq. (28),

$$K_0 = 15a_1 + \frac{9}{5}T + \frac{(\gamma - 5)(\gamma - 3)}{16} \rho_0^{\gamma/3} t_3. \quad (31)$$

If $t_3 = 0$, as $a_1 \sim 16$ MeV and $T \sim 37$ MeV are well-known from measurements, this formula gives the estimation $K_0 \sim 306$ MeV. Therefore, in order to have a K_0 value lower than 306 MeV, the fourth term $(\rho/\rho_0)^{\gamma/3}$ in the Skyrme EOS is needed.

3.2 Myers-Swiatecki interaction

For Myers-Swiatecki interaction, $D_\gamma^{\text{MS}}(\delta) = 0$, all the γ -dependent terms in eqs. (21)-(26) do not appear. The equilibrium condition (21) can be transformed into the following relation among α , B and $\bar{\gamma}$:

$$5\alpha - 10B - 4(1 - \bar{\gamma}) = 0, \quad (32)$$

where B and $\bar{\gamma}$ are defined, respectively, as

$$B_{l,u} = \frac{1}{2}(1 \mp \zeta)B, \quad \gamma_{l,u} = \frac{1}{2}(1 \mp \zeta)\bar{\gamma}. \quad (33)$$

Therefore, there are only 4 independent interaction parameters in the Myers-Swiatecki EOS, α , B , ξ and ζ , if $\bar{\gamma}$ is solved from eq. (32) as a function of α and B . Correspondingly, there are only 4 independent variables among a_1 , K_0 , J , L and K_s in the Myers-Swiatecki EOS. Actually, the following relationship can be derived:

$$\frac{K_s}{T} = \frac{4B(1 + \bar{\gamma})}{4B + \bar{\gamma}} \left[1 - \frac{10B + \bar{\gamma}}{2B(1 + \bar{\gamma})} \frac{3J - L}{T} \right], \quad (34)$$

where

$$B = \frac{5}{18} \frac{K_0 - 6a_1}{T}, \quad (35)$$

$$\bar{\gamma} = 1 - \frac{5}{9} \frac{K_0 - 15a_1}{T}. \quad (36)$$

Furthermore, formula (28) for the Myers-Swiatecki EOS becomes

$$K_0 = 15a_1 + \frac{9}{5}(1 - \bar{\gamma})T. \quad (37)$$

For $\bar{\gamma} = 0$, Myers-Swiatecki interaction is reduced to Seyler-Blanchard interaction [12] and this formula gives the estimation $K_0 \sim 306$ MeV, the same as that discussed for Skyrme interaction. Hence, $\bar{\gamma}$ -dependent terms in Myers-Swiatecki EOS are required, in order to obtain K_0 lower than 306 MeV [10].

3.3 Tondeur interaction

For Tondeur interaction, $D_5(\delta) = 0$, the term involving $D_5(0)$ in eq. (21) as well as all the terms involving D_{50} and D_{52} in eqs. (22)-(26) do not appear. The equilibrium condition (21) now is a relation among a , b and γ :

$$\frac{3\rho_0 a}{T} + \frac{\gamma\rho_0^{\gamma/3} b}{T} + \frac{6}{5} = 0. \quad (38)$$

In this case, there are only 3 independent interaction parameters in Tondeur EOS, for example a , c and γ . Correspondingly, there are only 3 free variables in a_1 , K_0 , J , L and K_s , for example a_1 , K_0 and J , since it can be shown that

$$L = 2J, \quad K_s = -2J. \quad (39)$$

In addition, the following relationship among K_0 , a_1 and γ can be written for Tondeur EOS from eq. (29):

$$K_0 = 3\gamma a_1 + \frac{3}{5}(\gamma - 2)T. \quad (40)$$

From $a_1 \sim 16$ MeV, $T \sim 37$ MeV and $K_0 \sim 220$ MeV, it can be evaluated that the appropriate integer is $\gamma = 4$, as given by Tondeur [4]. In this case, *i.e.*, if $\gamma = 4$ is chosen, there are only two interaction parameters to be freely adjusted in the data fit, for example a and c . Correspondingly, there are only two independent variables in a_1 , K_0 , J , L and K_s , for example a_1 and J , when K_0 is calculated by eq. (40). From $a_1 \sim 16$ MeV, $T \sim 37$ MeV and $\gamma = 4$ we can evaluate $K_0 \sim 236$ MeV. It is worthwhile to note that the value given by Tondeur is $K_0 = 235.8$ MeV [4].

The equilibrium condition is checked by calculating the expression on the left-hand side of eq. (21) for Skyrme, Myers-Swiatecki and Tondeur interactions, using the interaction parameters and the nuclear radius constant r_0 given in refs. [2], [3], and [4], respectively. These parameters will be referred to as the original interaction parameters thereafter. Besides, the following physical constants [13] are used in the present calculation: $\hbar c = 197.32891$ MeV · fm, $m = 938.90595$ MeV/ c^2 .

The calculated values are given as *EC* in the second column of table 1. It shows that the equilibrium condition of standard nuclear matter is fulfilled in the data fit to determine the original parameters of Skyrme (1st-5th row), Myers-Swiatecki (6th row), and Tondeur interactions (7th row), respectively.

The standard nuclear matter properties a_1 , K_0 , J , L , and K_s , calculated from Skyrme (1st-5th row), Myers-Swiatecki (6th row), and Tondeur interactions (7th row), respectively, are also given in the 5th-9th column of table 1, all in MeV. In this table, γ is a model parameter in eqs. (5) and (16) for Skyrme and Tondeur interactions, respectively, r_0 the nuclear radius constant used in the respective interaction, in fm.

As a comparison, the last two rows of table 1 (labeled by CWS) present the result obtained by fitting a_1 , K_0 , J , L , and K_s directly to nuclear masses [14]. It can be seen that these quantities have values close each other, except the case SIII, where the value of K_0 and K_s is far away from others. The average over the 2nd to 7th row gives $a_1 = 15.97$ MeV, $K_0 = 234.4$ MeV, $J = 29.25$ MeV, $L = 48.63$ MeV, and $K_s = -126.9$ MeV.

4 Nuclear matter away from the standard state

The nuclear matter state with zero pressure and minimum energy per nucleon can be solved from the following equation:

$$\frac{\partial e}{\partial \rho} = 0. \quad (41)$$

Usually there are several solutions, we should choose that one has minimum energy per nucleon. This solution gives

Table 1. The coefficients of volume energy a_1 , symmetry energy J , incompressibility K_0 , density symmetry L and symmetry incompressibility K_s calculated from the various Skyrme interactions [2](1st to 5th row), Myers-Swiatecki interaction [3](6th row), and Tondeur interaction [4](7th row), by original parameters, all in MeV. γ is a model parameter in eqs. (5) and (16) for Skyrme and Tondeur interactions, respectively. EC is the equilibrium criterion calculated from the left-hand side of eq. (21). As a comparison, the last two rows (labeled by CWS) present the result obtained by fitting these quantities directly to nuclear masses [14]. r_0 is the nuclear radius constant in fm.

EOS	EC	r_0	γ	a_1	K_0	J	L	K_s
SIII	0.00080	1.180	6	15.86	355.5	28.16	9.88	-393.9
Ska	-0.00001	1.154	4	15.99	263.1	32.91	74.62	-78.45
SkM	0.00004	1.142	7/2	15.77	216.6	30.75	49.34	-148.8
SkM*	0.00004	1.142	7/2	15.77	216.6	30.03	45.78	-155.9
RATP	0.00049	1.143	18/5	16.05	239.6	29.26	32.39	-191.3
M-S	0.00001	1.140		16.24	234.4	32.65	49.88	-147.1
Tondeur	0.00043	1.145	4	15.98	235.8	19.89	39.78	-39.78
CWS	0.00000	1.140	4	15.98	217.5	28.50	64.32	-101.3
CWS	0.00000	1.140	5	16.10	237.9	28.50	63.93	-114.2

density as function of δ :

$$\rho_m = \rho_m(\delta). \quad (42)$$

For $\delta = 0$, eq. (41) is reduced to the equilibrium condition of standard nuclear matter, we have

$$\rho_m(0) = \rho_0. \quad (43)$$

The incompressibility of non-equilibrium nuclear matter, which is of interest in many applications, can be defined as [11]

$$K(\rho, \delta) = 9 \frac{\partial P}{\partial \rho}, \quad (44)$$

where $P = \rho^2 \partial e / \partial \rho$ is the pressure. Along the line of minimum (42), this $K(\rho, \delta)$ becomes

$$K_m(\delta) = 9 \left[\rho^2 \frac{\partial^2 e}{\partial \rho^2} \right]_{\rho=\rho_m}. \quad (45)$$

At the standard state $(\rho_0, 0)$ we have $K_m(0) = K_0$. At the critical point (ρ_c, δ_c) , where the maximum and the minimum are coincident, the curvature of $e(\rho, \delta_c)$ vs. ρ changes sign and $K_m(\delta_c) = 0$. So $K_m(\delta)$ starts with K_0 and ends at 0 when δ increases along the line of minimum. In addition, the generalized symmetry energy of non-equilibrium nuclear matter can be defined as [6] [1] [9]

$$J(\rho) = \frac{1}{2} \frac{\partial^2 e}{\partial \delta^2} \Big|_{\delta=0}. \quad (46)$$

In term of this quantity, the usual symmetry energy J can be expressed as

$$J = J(\rho_0). \quad (47)$$

For nuclear matter not far away from the standard state $(\rho_0, 0)$, the EOS can be written approximately as [14]

$$e(\rho, \delta) \approx -a_1 + \frac{1}{18} (K_0 + K_s \delta^2) \left(\frac{\rho - \rho_0}{\rho_0} \right)^2 + \left[J + \frac{L}{3} \left(\frac{\rho - \rho_0}{\rho_0} \right) \right] \delta^2. \quad (48)$$

In this approximation, we have

$$K(\rho, \delta) \approx (K_0 + K_s \delta^2) \left(\frac{\rho}{\rho_0} \right)^2, \quad (49)$$

$$J(\rho) \approx J + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0} + \frac{K_s}{18} \left(\frac{\rho - \rho_0}{\rho_0} \right)^2. \quad (50)$$

Using eq. (48), the following solutions can be obtained:

$$\rho_m \approx \rho_0 \left(1 - \frac{3L}{K_0} \delta^2 \right), \quad (51)$$

$$e_m \approx -a_1 + J \delta^2, \quad (52)$$

$$K_m \approx K_0 + K_s \delta^2, \quad (53)$$

where only the linear term in δ^2 is kept. The systematics of nuclear central densities [15] based on elastic electron scattering data [17,18] and muonic atom spectroscopy data [18] provide a direct evidence for eq. (51).

Thus, in the plot $e(\rho, \delta)$ vs. ρ , we have the geometric meaning of a_1 , K_0 , J , L , and K_s : the standard state is at the minimum point $\rho_m = \rho_0$ with depth a_1 and curvature proportional to K_0 ; when the minimum is moved with increasing δ from 0, the decrease of ρ_m is controlled by $3L/K_0$, the increase of depth is controlled by J , while the decrease of curvature is controlled by $-K_s$. Therefore, the quantities a_1 , K_0 , J , L , and K_s are characteristics of nuclear matter not only at standard state but also at the state not far away from the standard one. In this way, the interaction with different value of these quantities will predict different properties of nuclear matter that are not far away from the standard state.

The exact solution $\rho_m(\delta)$ depends on the interaction. The analytic solution is possible for SIII, Ska, Myers-Swiatecki, and Tondeur interactions, while the numerical solution is appropriate for SkM, SkM*, and RATP interactions.

Table 2. Input values used to readjust the interaction parameters, r_0 in fm, others in MeV.

Force	r_0	a_1	K_0	J	L	K_s
Skyrme	1.140	15.97	236.07	29.25	58.50	-67.92
M-S	1.140	15.97	236.07	29.25	58.50	-67.92
Tondeur	1.140	15.97	236.07	29.25	58.50	-58.50

For Myers-Swiatecki interaction, (41) is a cubic equation which gives

$$\left(\frac{\rho_m}{\rho_0}\right)^{1/3} = 2s_0 \sin\left(\frac{\pi}{6} + \frac{\theta}{3}\right), \quad (54)$$

where

$$s_0 = \left[\frac{1}{5} \frac{D_3(\delta)}{D_5(\delta)}\right]^{1/2}, \quad \cos\theta = \frac{1}{5s_0^3} \frac{D_2(\delta)}{D_5(\delta)}. \quad (55)$$

The superscript *MS* for Myers-Swiatecki's D is dropped for simplicity. The critical δ_c , where the maximum and the minimum is coincident, is determined by

$$D_2(\delta_c) = 5s_0^3 D_5(\delta_c), \quad (56)$$

which corresponds to $\theta = 0$ and

$$\rho_c = \rho_m(\delta_c) = s_0^3 \rho_0. \quad (57)$$

For Tondeur interaction with $\gamma = 4$, (41) is a quadratic equation which gives

$$\left(\frac{\rho_m}{\rho_0}\right)^{1/3} = \frac{1}{8D_4} \{3D_3 + [9D_3^2 - 32D_4 D_2(\delta)]^{1/2}\}, \quad (58)$$

where D_3 and D_4 are numbers, the superscript *To* for Tondeur's D is dropped also. The critical point is given by

$$9D_3^2 - 32D_4 D_2(\delta_c) = 0, \\ \rho_c = \left(\frac{3D_3}{8D_4}\right)^3 \rho_0 = -\left(\frac{3a}{8b}\right)^3. \quad (59)$$

As the location ρ_0 and depth a_1 are different for different equation of state, as shown in table 1, a way to make comparison is to plot the normalized energy per nucleon e/a_1 as a function of the relative nucleon density ρ/ρ_0 for given δ . Figure 1a shows this e/a_1 vs. ρ/ρ_0 for $\delta = 0$, calculated by the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). The solid lines in the right-hand side of the plot, from top to bottom, correspond to SIII, Ska, RATP, SkM, and SkM* interactions, respectively. The difference between SkM and SkM* is negligible and Myers-Swiatecki is almost coincident with RATP. This sequence is just the decreasing sequence of K_0 's value, as shown in table 1: the smaller value of K_0 , the smaller curvature of the curve at the standard state, thus the softer the EOS.

A natural question is: what is the difference among these EOS's, if the standard state is the same with same

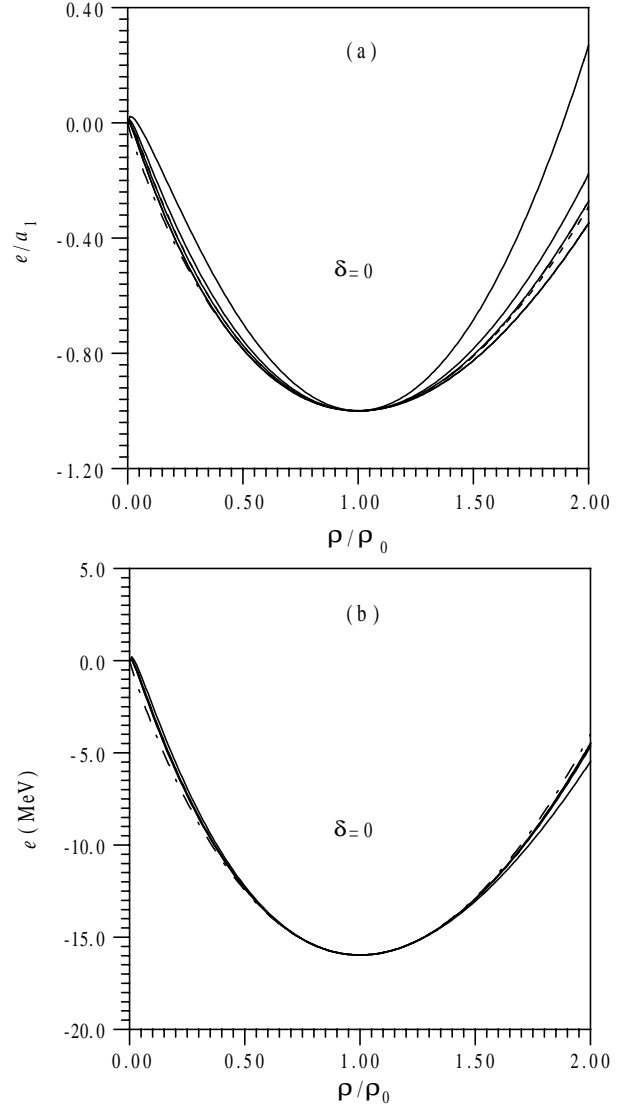


Fig. 1. (a) Normalized energy per nucleon e/a_1 vs. relative nucleon density ρ/ρ_0 calculated by original interaction parameters at $\delta = 0$, (b) e vs. ρ/ρ_0 calculated by readjusted interaction parameters at $\delta = 0$ of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). The solid lines in the right-hand side of the plot in (a) from top to bottom correspond to SIII, Ska, RATP, SkM, and SkM* interactions, respectively. The difference between SkM and SkM* is negligible and Myers-Swiatecki is almost coincident with RATP.

location ρ_0 , depth a_1 , curvature $\sim K_0$ and so on? In order to make this comparison, the interaction parameters should be readjusted according to chosen ρ_0 , a_1 , K_0 and so on. For the value of ρ_0 , we choose $r_0 = 1.140$ fm which is well determined by the data fit to nuclear charge radii [19] extracted from elastic electron scattering data [17]. In addition, we can choose the value of a_1 , K_0 , J , L , and K_s in an appropriate way. In this case, eq. (34) should be fulfilled for Myers-Swiatecki interaction, while eqs. (39) and (40) should be fulfilled for Tondeur interaction. The

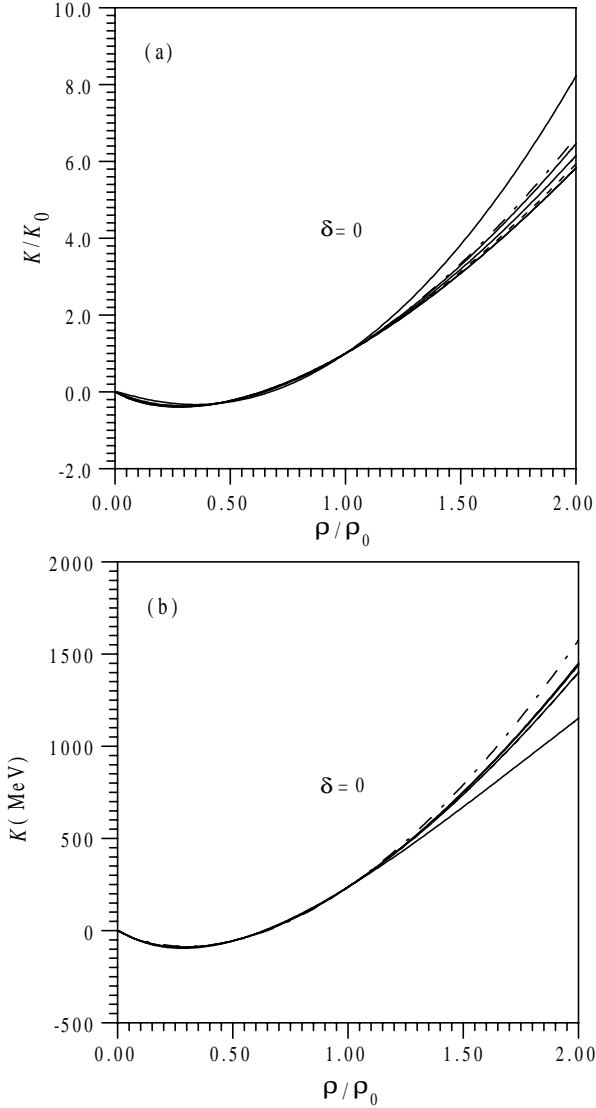


Fig. 2. (a) Normalized nuclear incompressibility K/K_0 vs. relative nucleon density ρ/ρ_0 calculated by original interaction parameters at $\delta = 0$, (b) K vs. ρ/ρ_0 calculated by readjusted interaction parameters at $\delta = 0$ of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). On the right-hand side of the plot in (a) the solid lines from top to bottom correspond to SIII, Ska, RATP, SkM, and SkM* interactions, respectively. The difference between SkM and SkM* is negligible. In (b), the solid lines from top to bottom are due to SkM, SkM*, RATP, Ska, and SIII, where SkM* is identical to SkM. Tondeur is coincident with Ska; SkM and RATP almost overlap.

chosen values used to calculate the interaction parameters are listed in table 2, while the formulas used to perform this calculation are given in Appendix B. Among these values, a_1 and J are the average values given in the last section, K_0 and L are calculated by eqs. (40) and (39). K_s is calculated by eq. (39) in Tondeur's case while by eq. (34) in Myers-Swiatecki's case. In Skyrme's case, K_s can be chosen from either Myers-Swiatecki's or Tondeur's

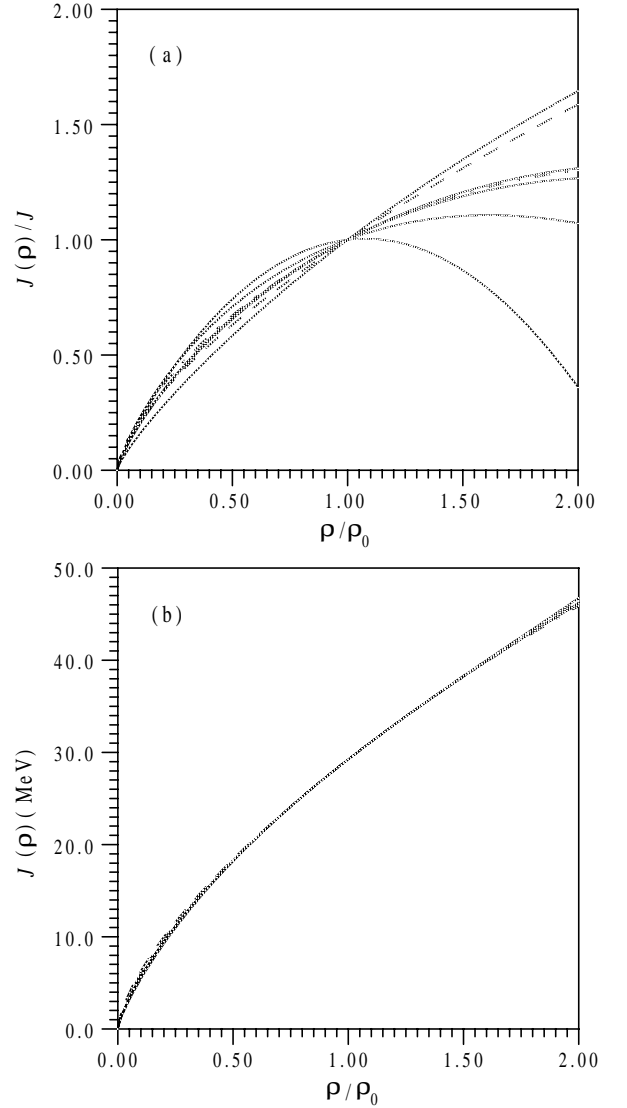


Fig. 3. (a) Normalized symmetry energy $J(\rho)/J$ vs. the relative nucleon density ρ/ρ_0 calculated by original interaction parameters, (b) $J(\rho)$ vs. ρ/ρ_0 calculated by readjusted interaction parameters of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). On the right-hand side of the plot, in (a), the solid lines, from top to bottom, correspond to Ska, SkM, SkM*, RATP, and SIII interactions, respectively. The difference between SkM and SkM* for $\rho/\rho_0 < 1.3$ is negligible. In addition, Myers-Swiatecki's interaction is almost coincident with SkM's. In (b), the difference among these curves is negligible.

value, there is no significant difference in the calculated result which will be shown in the following.

The calculated Skyrme interaction parameters are given in table 3. The Myers-Swiatecki interaction parameters are calculated as

$$\begin{aligned} \alpha &= 2.06285, & B &= 1.05232, & \bar{\gamma} &= 1.05222, \\ \xi &= 0.12333, & \zeta &= 0.37363. \end{aligned} \quad (60)$$

Table 3. Readjusted Skyrme interaction parameters t_0 , t_3 , x_0 , s_1 , and s_2 . Input values are $r_0 = 1.140$ fm, $a_1 = 15.97$ MeV, $K_0 = 236.07$ MeV, $J = 29.25$ MeV, $L = 58.50$ MeV, and $K_s = -67.92$ MeV.

Force	SIII	Ska	SkM	SkM*	RATP
γ	6	4	7/2	7/2	18/5
t_0 (MeV fm ³)	-1405.521	-1792.320	-2372.518	-2372.518	-2179.119
t_3 (MeV fm ⁷)	-14402.55	12794.56	12584.33	12584.33	11940.95
x_0	0.06956	0.13735	0.19759	0.19759	0.18018
x_3	0.38368	0.38368	0.38368	0.38368	0.38368
s_1 (MeV fm ⁵)	642.825	-42.389	71.813	71.813	55.499
s_2 (MeV fm ⁵)	-473.186	84.802	-8.196	-8.196	5.090

Table 4. Critical point (ρ_c, δ_c) predicted by Skyrme, Myers-Swiatecki, and Tondeur interactions, respectively. r_0 in fm, ρ_c in fm⁻³, and e_c in MeV. For each item, the first line is given by original interaction parameters, the second line by readjusted parameters shown in table 3 for Skyrme interactions while by eqs. (60) and (61) for Myers-Swiatecki and Tondeur interactions, respectively.

	SIII	Ska	SkM	SkM*	RATP	M-S	Tondeur
r_0	1.180	1.154	1.142	1.142	1.143	1.140	1.145
	1.140	1.140	1.140	1.140	1.140	1.140	1.140
δ_c	0.8385	0.8647	0.8390	0.8421	0.8303	0.8213	0.8732
	0.8772	0.8980	0.8908	0.8908	0.8920	0.8988	0.7697
ρ_c	0.07173	0.02416	0.02345	0.02420	0.03892	0.03039	0.03081
	0.02732	0.02969	0.02825	0.02825	0.02851	0.02643	0.03131
e_c	3.9019	1.5852	1.2572	1.2814	1.9898	1.1031	2.6142
	1.8894	1.8505	1.7025	1.7025	1.7311	1.1280	2.6304

For $\gamma = 4$, Tondeur interaction parameters are calculated as

$$\begin{aligned} a &= -672.13 \text{ MeV fm}^3, \quad b = 799.71 \text{ MeV fm}^4, \\ c &= 99.116 \text{ MeV fm}^2. \end{aligned} \quad (61)$$

These parameters will be referred to as the readjusted interaction parameters thereafter.

As a comparison with the result calculated by original interaction parameters, fig. 1b plots e vs. ρ/ρ_0 for $\delta = 0$, calculated by readjusted interaction parameters. It can be seen that now there is almost no difference among these EOS's for $0.4 < \rho/\rho_0 < 1.6$.

Figure 2 displays in (a) the normalized nuclear incompressibility K/K_0 vs. the relative nucleon density ρ/ρ_0 calculated by original interaction parameters at $\delta = 0$, and in (b) K vs. ρ/ρ_0 calculated by readjusted interaction parameters at $\delta = 0$ of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). Considering the right-hand side of the plot, in (a), the solid lines, from top to bottom, correspond to SIII, Ska, RATP, SkM, and SkM* interactions, respectively. The difference between SkM and SkM* is negligible. In (b), the solid lines, from top to bottom, are due to SkM, SkM*, RATP, Ska, and SIII, where SkM* is identical to SkM. Tondeur is coincident with Ska; SkM and RATP almost overlap. It can be seen from this figure that the difference among these curves is negligible for $\rho/\rho_0 < 1.2$. This is expected from eq. (49) which shows that the curve is determined essentially by K_0 .

Figure 3 depicts in (a) the normalized symmetry energy $J(\rho)/J$ vs. the relative nucleon density ρ/ρ_0 calculated by original interaction parameters, in (b) $J(\rho)$

vs. ρ/ρ_0 calculated by readjusted interaction parameters of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). Considering the right-hand side of the plot, in (a), the solid lines, from top to bottom, correspond to Ska, SkM, SkM*, RATP, and SIII interactions respectively. The difference between SkM and SkM* for $\rho/\rho_0 < 1.3$ is negligible. In addition, Myers-Swiatecki's interaction is almost coincident with SkM's. In (b), the difference among these curves is negligible. This is expected from eq. (50), which shows that, for the density ρ is not far away from ρ_0 , the symmetry energy $J(\rho)$ is determined essentially by J , L and K_s , and these quantities (J and L) are the same or almost the same (K_s) for the readjusted interaction parameters.

Using the interaction parameters, we can calculate $\rho_m(\delta)$, $e_m(\delta)$, and $K_m(\delta)$ along the equilibrium line. The result is shown in figs. 4-6, while the critical point value (ρ_c, δ_c) is listed in table 4, for Skyrme, Myers-Swiatecki, and Tondeur interactions, respectively.

Figure 4 shows the equilibrium density ρ_m as function of the relative neutron excess δ , calculated by (a) original parameters and (b) readjusted parameters of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). The solid lines, in the middle range of δ , from top to bottom, in (a), correspond to SIII, RATP, SkM, SkM*, and Ska interactions respectively, the difference between SkM and SkM* is very small. In (b), the solid lines from top to bottom correspond to Ska, RATP, SkM, SkM*, and SIII interactions, respectively. SkM and SkM* are the

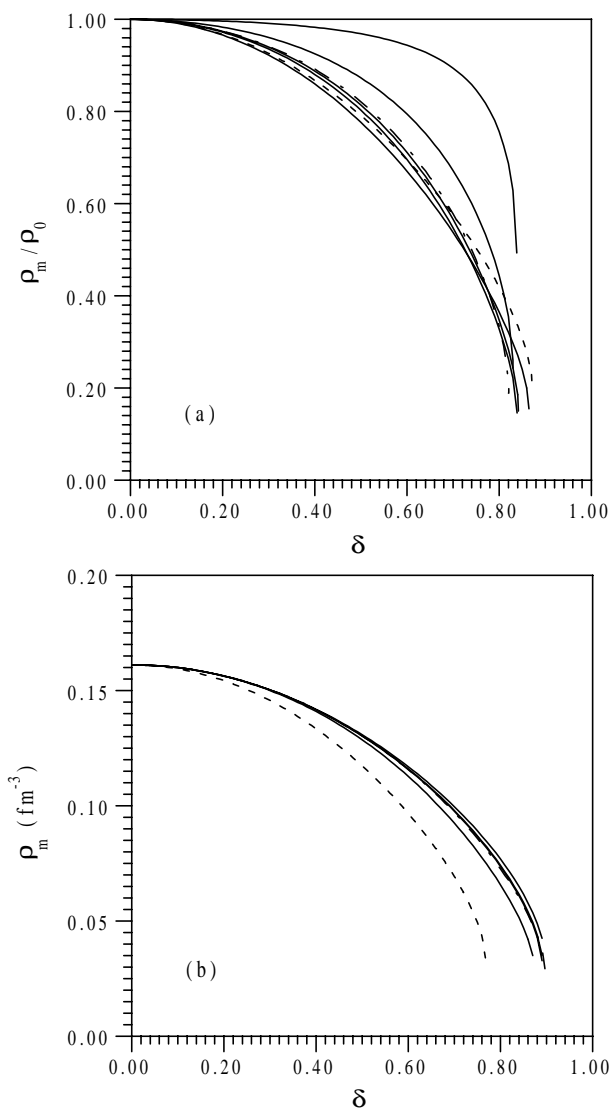


Fig. 4. Equilibrium density ρ_m as function of the relative neutron excess δ , calculated by (a) original parameters and (b) readjusted parameters of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). The solid lines, in the middle range of δ from top to bottom in (a) correspond to SIII, RATP, SkM, SkM*, and Ska interactions, respectively, the difference between SkM and SkM* is very small. In (b) the solid lines from top to bottom correspond to Ska, RATP, SkM, SkM*, and SIII interactions, respectively. SkM and SkM* are the same whereas RATP, SkM and Myers-Swiatecki almost overlap.

same whereas RATP, SkM and Myers-Swiatecki almost overlap.

Figure 5 gives the equilibrium energy per nucleon e_m as function of the relative neutron excess δ , calculated by (a) original parameters and (b) readjusted parameters of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). The solid lines, on the right-hand side, in (a), correspond to RATP, SkM, SkM*, SIII, and Ska interactions, from top to bottom, respectively. SIII, Ska and Tondeur almost overlap, whereas the difference between SkM and SkM* is negligible. In (b) the solid lines from top to bottom correspond to SIII, SkM, SkM*, RATP, and Ska interactions, respectively, where SkM and SkM* are the same. SkM, RATP and Ska are almost coincident.

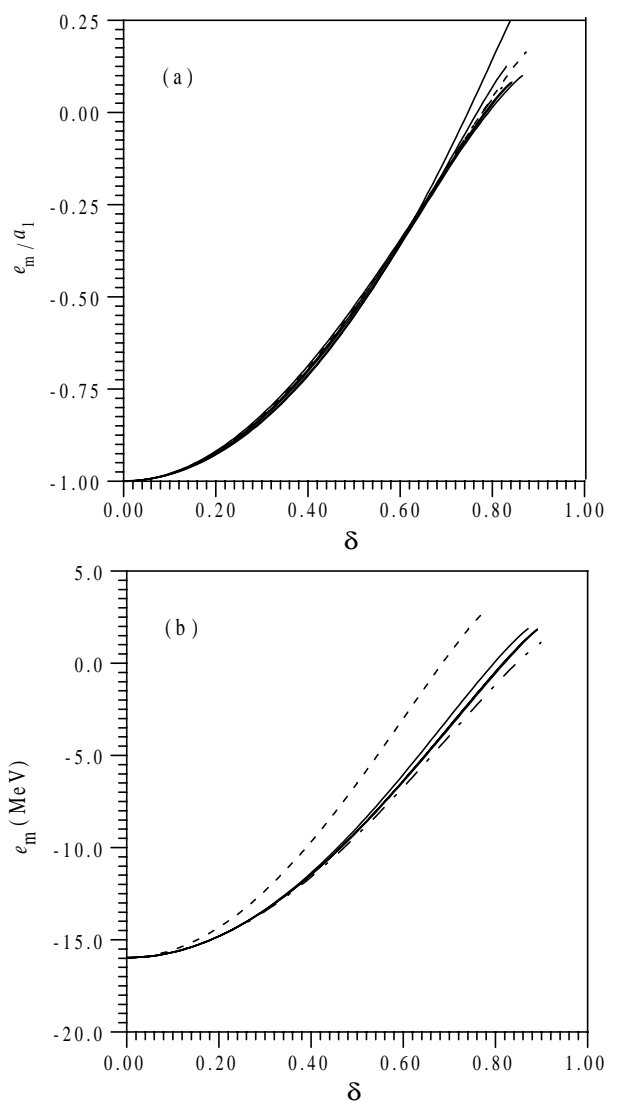


Fig. 5. Equilibrium energy per nucleon e_m as function of the relative neutron excess δ , calculated by (a) original parameters and (b) readjusted parameters of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). The solid lines, on the right-hand side, in (a) correspond to RATP, SkM, SkM*, SIII, and Ska interactions, from top to bottom, respectively. SIII, Ska and Tondeur almost overlap, whereas the difference between SkM and SkM* is negligible. In (b) the solid lines from top to bottom correspond to SIII, SkM, SkM*, RATP, and Ska interactions, respectively, where SkM and SkM* are the same. SkM, RATP and Ska are almost coincident.

Ska and Tondeur almost overlap, whereas the difference between SkM and SkM* is negligible. In (b), the solid lines from top to bottom correspond to SIII, SkM, SkM*, RATP, and Ska interactions, respectively, where SkM and SkM* are the same. SkM, RATP and Ska are almost coincident.

Figure 6 plots the equilibrium incompressibility K_m as function of the relative neutron excess δ , calculated by (a) original parameters and (b) readjusted parameters

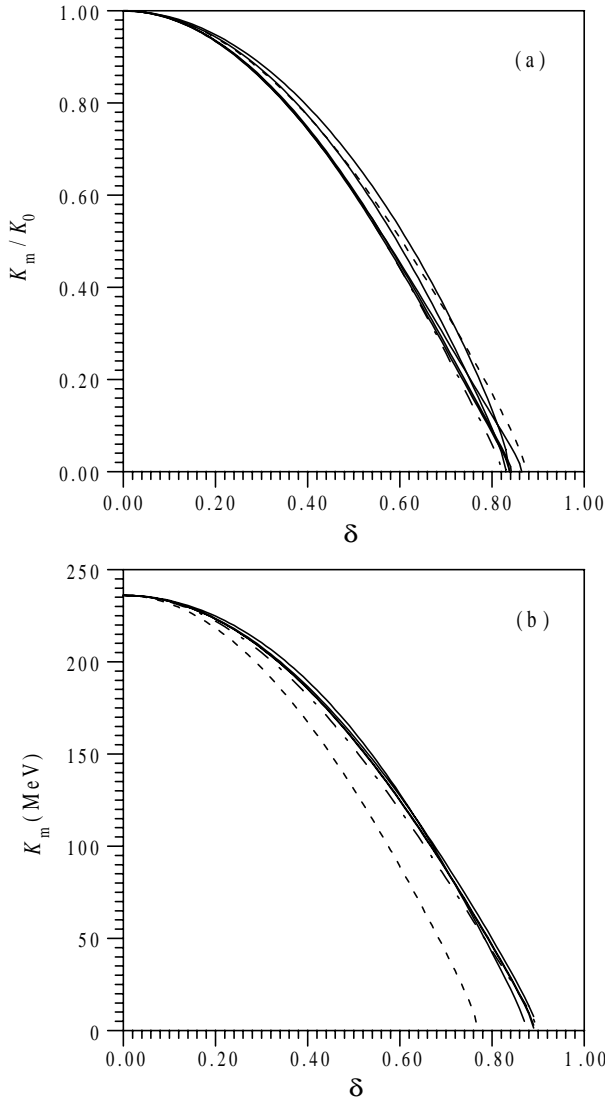


Fig. 6. Equilibrium incompressibility K_m as function of the relative neutron excess δ , calculated by (a) original parameters and (b) readjusted parameters of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). The solid lines in (a) correspond to SIII, RATP, Ska, SkM*, and SkM interactions from top to bottom in the middle range of δ , respectively. Ska, SkM* and SkM almost overlap. In (b) the solid lines from top to bottom correspond to Ska, RATP, SkM, SkM*, and SIII interactions, respectively; SkM and SkM* are the same.

of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). The solid lines in (a) correspond to SIII, RATP, Ska, SkM*, and SkM interactions, from top to bottom in the middle range of δ , respectively. Ska, SkM* and SkM almost overlap. In (b), the solid lines, from top to bottom, correspond to Ska, RATP, SkM, SkM*, and SIII interactions, respectively; SkM and SkM* are the same.

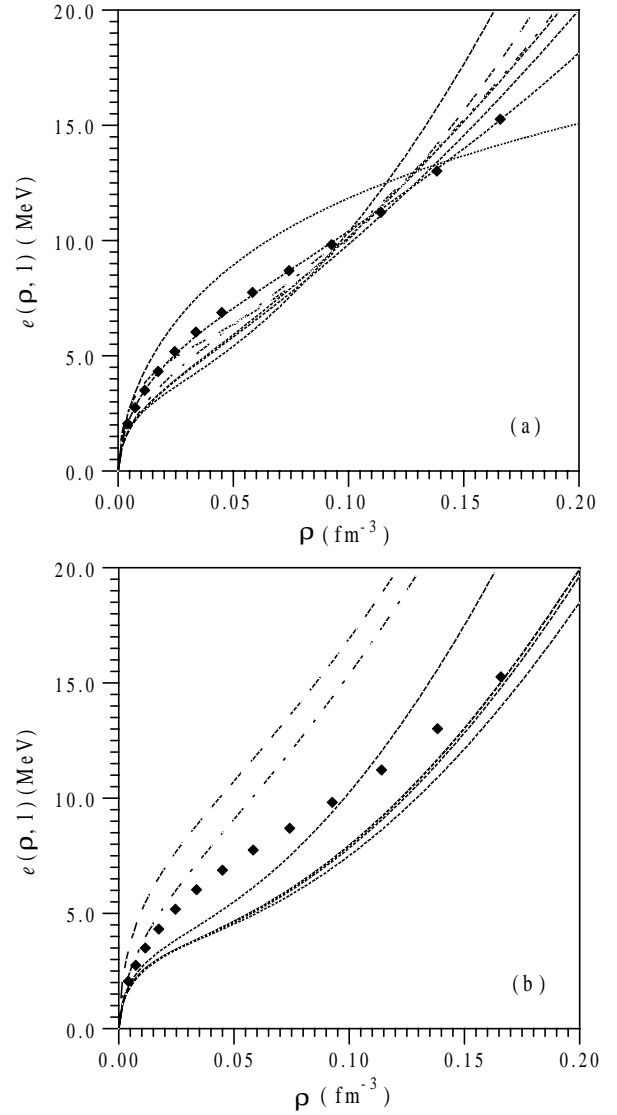


Fig. 7. Neutron matter EOS $e(\rho, 1)$ vs. nucleon density ρ , calculated by (a) original parameters and (b) readjusted parameters of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). Black diamonds denote the data taken from ref. [5]. The solid lines, on the right-hand side of (a), correspond to Ska, SkM, SkM*, RATP, and SIII from top to bottom. In (b), the solid lines, from top to bottom, correspond to SIII, SkM, SkM*, RATP, and Ska, where SkM and SkM* are identical.

5 Comparison with microscopic calculations

In order to provide additional elements about the confidence on the effective interactions discussed above, it is interesting to make a comparison with some microscopic calculations which are based on a more fundamental level of theories as well as on very different physical input. In fig. 7 the present predictions for the pure neutron matter EOS are compared with the theoretical estimates of Friedman and Pandharipande [5], obtained from a variational framework based on the Urbana v_{14} two-nucleon poten-

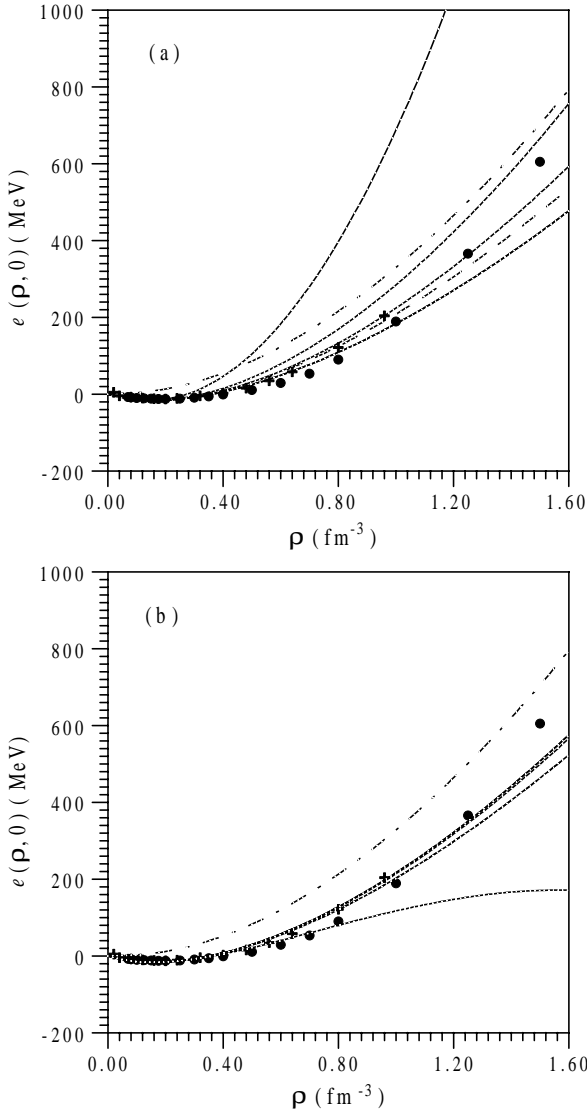


Fig. 8. Symmetric nuclear matter EOS $e(\rho, 0)$, calculated by (a) original parameters and (b) readjusted parameters of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). The full dots stand for the results taken from ref. [6], the crosses denote the results taken from ref. [8]. The solid lines, on the right-hand side of (a), correspond to SIII, Ska, RATP, SkM, and SkM*, from top to bottom, where SkM and SkM* are identical. In (b), the solid lines, from top to bottom, correspond to SkM, SkM*, RATP, Ska, and SIII, where SkM and SkM* are identical. The Tondeur's is very close to the Ska's.

tial plus three-nucleon interaction model of Lagaris and Pandharipande [20]. The neutron matter EOS $e(\rho, 1)$ vs. nucleon density ρ is calculated by (a) original parameters and (b) readjusted parameters of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). Black diamonds denote the data taken from ref. [5]. The solid lines, on the right-hand side of (a), correspond to Ska, SkM, SkM*, RATP, and SIII from top to bottom. In (b),

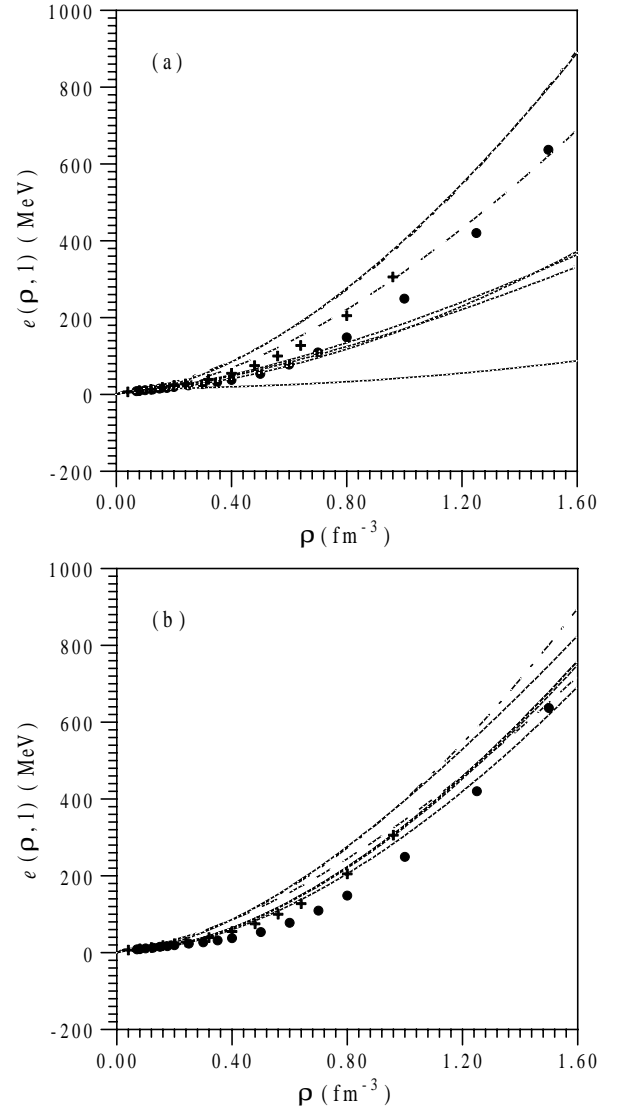


Fig. 9. Neutron matter EOS $e(\rho, 1)$, calculated by (a) original parameters and (b) readjusted parameters of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). The full dots stand for the results taken from ref. [6], the crosses denote the results taken from ref. [8]. The solid lines, on the right-hand side of (a), correspond to Ska, RATP, SkM, SkM*, and SIII, from top to bottom. In (b), the solid lines, from top to bottom, correspond to SIII, SkM, SkM*, RATP, and Ska, where SkM and SkM* are identical.

the solid lines, from top to bottom, correspond to SIII, SkM, SkM*, RATP, and Ska, where SkM and SkM* are identical. It seems that in the low density region the neutron matter EOS's calculated by original parameters are closer to microscopic results than those by readjusted parameters. However, the situation is different if the density is extended to higher region (see fig. 9).

It is worthwhile to note that the generalized Skyrme interaction FPS21 proposed by Pethick, Ravenhall and Lorenz [7] has the property that it is a good fit to both the nuclear and neutron matter calculations of Friedman and

Pandharipande. In this sense, fig. 7 may be regarded also as a comparison between our results and those of FPS21.

Even the EOS's based on effective interactions and energy functional theories discussed in the present work are essentially nonrelativistic, it is still interesting to see how they behave in the high density region. Figure 8 gives the symmetric nuclear matter EOS's $e(\rho, 0)$ up to about $10\rho_0$, calculated by (a) original parameters and (b) readjusted parameters of the various Skyrme interactions (solid lines), Myers-Swiatecki interaction (dot-dashed line), and Tondeur interaction (dashed line). The full dots stand for the results taken from ref. [6], which is a microscopic calculation of EOS for dense nuclear and neutron matter based on the Argonne v_{14} two-nucleon potential plus Urbana VII three-nucleon potential. The crosses denote the results taken from ref. [8], which studied the properties of dense nucleon matter and the structure of neutron stars, using variational chain summation methods and the new Argonne v_{18} two-nucleon interaction and the Urbana model IX of three-nucleon interaction as well as the relativistic boost correction to the two-nucleon interaction. The solid lines, on the right-hand side of (a), correspond to SIII, Ska, RATP, SkM, and SkM*, from top to bottom, where SkM and SkM* are identical. In (b), the solid lines, from top to bottom, correspond to SkM, SkM*, RATP, Ska, and SIII, where SkM and SkM* are identical. Tondeur's interaction is very close to the Ska's.

Figure 9 is the same as fig. 8 but for pure neutron matter EOS $e(\rho, 1)$. The solid lines, on the right-hand side of (a), correspond to Ska, RATP, SkM, SkM*, and SIII, from top to bottom. In (b), the solid lines, from top to bottom, correspond to SIII, SkM, SkM*, RATP, and Ska, where SkM and SkM* are identical.

6 Discussion and Summary

A discussion of the nuclear matter EOS's based on Skyrme, Myers-Swiatecki and Tondeur interactions is given in this paper. The equations are in the form of polynomials in the cubic root of density, with coefficients that are functions of the relative neutron excess and depend on the model of interaction.

Most of the discussion about the nuclear EOS, up to now, focus at states around standard state, *i.e.* about the quantities a_1 , J , L , K_0 , and K_s ; especially K_0 in supernova explosion and neutron star calculations and K_s in heavy-ion collisions. However, even these quantities or equivalently the interaction parameters were well determined by the measured data of nuclei, mainly the nuclear masses, the extrapolation to states far away from standard state is still an open problem. It is seen that the difference among these EOSs is not significant in most of the relative neutron excess range which is of interest for both heavy-ion collisions and supernova explosion calculations. However, if the equations are fitted to the same standard state, the equation based on Tondeur interaction is softer than others provided the relative neutron excess is not close to 0 [16].

The numerical result given in section 4 shows that the asymmetry dependence of the critical point depends on the model used in the extrapolation. When the EOS is fitted to same standard state, Skyrme's and Myers-Swiatecki's δ_c are close each other, especially δ_c does not depend sensitively on the choice of γ in Skyrme interaction. On the other hand, Tondeur's δ_c is much smaller than others. This is because the value of Tondeur's δ_c depends sensitively on the interaction parameters, as it can be seen and checked numerically from the first equation of (59). In this content, in order to make a choice among these interactions for the extrapolation, experiments which can provide direct or even indirect information about nuclear matter with large asymmetry δ and low density ρ are required.

The authors acknowledge the support from the Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ). C.S.W. and J.W.Z. also acknowledge the partial support by the National Natural Science Foundation of China and the Special Fund of China for the Universities with Ph.D. Programs.

Appendix A.

It will be shown here that the energy density functional \mathcal{E}_{GD} of Myers-Swiatecki interaction can be written approximately in the form of eq. (3). In the Thomas-Fermi model of nuclei and up to the second order of the localized approximation given in ref. [10], we have

$$\mathcal{E}_{\text{GD}}^{\text{MS}} = aI_1(r/a)F^{(1)}(r) + \frac{a^2}{2}I_2(r/a)F^{(2)}(r), \quad (\text{A.1})$$

where a is the Yukawa range of force,

$$I_1(x) = \frac{2}{x}(1 - e^{-x}), \quad I_2(x) = 2(1 + 2e^{-x}), \quad (\text{A.2})$$

$$F^{(1)}(r) = T \left[\epsilon_{1n} \frac{d\rho_n}{dr} + \epsilon_{1p} \frac{d\rho_p}{dr} \right], \quad (\text{A.3})$$

$$F^{(2)}(r) = T \left[\epsilon_{1n} \frac{d^2\rho_n}{dr^2} + \epsilon_{1p} \frac{d^2\rho_p}{dr^2} + \frac{2\epsilon_{2n}}{\rho_0} \left(\frac{d\rho_n}{dr} \right)^2 + \frac{2\epsilon_{2p}}{\rho_0} \left(\frac{d\rho_p}{dr} \right)^2 \right]. \quad (\text{A.4})$$

In the above two equations, $\epsilon_{1n}, \epsilon_{1p}, \epsilon_{2n}$ and ϵ_{2p} are the functionals of nucleon densities $\rho_n(r)$ and $\rho_p(r)$ whose specific expressions are given in ref. [10]. Using the approximation of $I_1(x) \approx 2/x$ and $I_2(x) \approx 2$ which are explained and employed in ref. [10], the following result can be obtained:

$$\mathcal{E}_{\text{GD}}^{\text{MS}} = a^2 T (\epsilon_{1n} \nabla^2 \rho_n + \epsilon_{1p} \nabla^2 \rho_p). \quad (\text{A.5})$$

In the simplified Myers-Swiatecki interaction, we have [10]

$$\begin{aligned} \epsilon_{1n} &= - \left[\alpha_t \frac{\rho_n}{\rho_0} + \alpha_u \frac{\rho_p}{\rho_0} \right], \\ \epsilon_{1p} &= - \left[\alpha_t \frac{\rho_p}{\rho_0} + \alpha_u \frac{\rho_n}{\rho_0} \right], \quad \epsilon_{2n} = \epsilon_{2p} = 0, \end{aligned} \quad (\text{A.6})$$

where $\alpha_{l,u} = \frac{1}{2}(1 \pm \xi)\alpha$, thus the functional $\mathcal{E}_{\text{GD}}^{\text{MS}}$ can be reduced to

$$\mathcal{E}_{\text{GD}}^{\text{MS}} = \frac{a^2 T}{\rho_0} \{ \alpha_u (\nabla \rho)^2 + (\alpha_l - \alpha_u) [(\nabla \rho_n)^2 + (\nabla \rho_p)^2] \}. \quad (\text{A.7})$$

For the symmetric case with $\rho_n = \rho_p = \rho/2$, we have finally

$$\mathcal{E}_{\text{GD}}^{\text{MS}} = \frac{a^2 T}{2\rho_0} \alpha (\nabla \rho)^2. \quad (\text{A.8})$$

Appendix B.

The formulas to calculate the interaction parameters from the nuclear matter quantities a_1 , K_0 , J , L , and K_s will be given here for Skyrme, Myers-Swiatecki, and Tondeur interactions, respectively. In Skyrme interaction, s_1 , s_2 , t_3 , and x_3 can be calculated by the following equations:

$$s_1 + \frac{1}{2}s_2 = \left(\frac{2}{3\pi^2} \right)^{2/3} \frac{5}{6(\gamma-5)} \times \left[\frac{3(\gamma-2)}{5} + \frac{3\gamma a_1 - K_0}{T} \right] \frac{T}{\rho_0^{5/3}}, \quad (\text{B.1})$$

$$s_1 + 2s_2 = \left(\frac{2}{3\pi^2} \right)^{2/3} \frac{3}{2(\gamma-5)} \times \left[\frac{\gamma-2}{3} - \frac{\gamma(3J-L) + K_s}{T} \right] \frac{T}{\rho_0^{5/3}}, \quad (\text{B.2})$$

$$t_3 = \frac{16}{(\gamma-5)(\gamma-3)} \left[\frac{K_0 - 15a_1}{T} - \frac{9}{5} \right] \frac{T}{\rho_0^{7/3}}, \quad (\text{B.3})$$

$$x_3 = \frac{3T - 5(3J-L) - K_s}{2K_0 - 15a_1 - \frac{9}{5}T} - \frac{1}{2}. \quad (\text{B.4})$$

Having s_1 , s_2 , and t_3 , t_0 can be calculated by (30). Finally, x_0 can be calculated by

$$t_0(x_0 + \frac{1}{2}) = \frac{2}{\gamma-3} \left[\gamma-2 - \frac{5\gamma J - (\gamma+2)L + K_s}{T} \right] \frac{T}{\rho_0}. \quad (\text{B.5})$$

Myers-Swiatecki interaction parameters can be calculated as

$$\alpha = \frac{K_0 - 10a_1}{T}, \quad (\text{B.6})$$

$$B = \frac{5}{18} \frac{K_0 - 6a_1}{T}, \quad (\text{B.7})$$

$$\bar{\gamma} = 1 - \frac{5}{9} \frac{K_0 - 15a_1}{T}, \quad (\text{B.8})$$

$$\xi = -\frac{4B(1+\bar{\gamma})}{\alpha(4B+\bar{\gamma})} \left[1 - \frac{5B-\bar{\gamma}}{B(1+\bar{\gamma})} \frac{J}{T} + \frac{2B-\bar{\gamma}}{2B(1+\bar{\gamma})} \frac{L}{T} \right], \quad (\text{B.9})$$

$$\zeta = \frac{1}{3} - \frac{2(1+\bar{\gamma})}{3(4B+\bar{\gamma})} \left[1 - \frac{3}{1+\bar{\gamma}} \frac{3J-L}{T} \right]. \quad (\text{B.10})$$

Tondeur interaction parameters are

$$a = -\frac{3\gamma-2}{5\gamma-3} \left(1 + \frac{5}{3} \frac{\gamma}{\gamma-2} \frac{a_1}{T} \right) \frac{T}{\rho_0}, \quad (\text{B.11})$$

$$b = \frac{3}{\gamma-3} \frac{1}{\rho_0^{7/3}} \left(a_1 + \frac{T}{5} \right), \quad (\text{B.12})$$

$$c = \frac{J}{\frac{2}{3}\rho_0}. \quad (\text{B.13})$$

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